2/28/17

"Its easy to be happy when people do what they're supposed to."- Chris Callahan

HW: "Direct Variation" Homework section

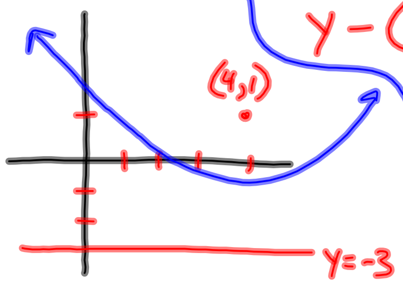
AIM: What is Direct Variation?

Warm Up:

Determine the equation of the parabola whose focus is the point $(4, 1)$ and whose directrix is the horizontal line $y = -3$. First, draw a diagram that shows the parabola, then carefully use the distance formula to derive its equation.

Distance to the Directrix = Distance to the Focus

$y - (\text{Equation of Directrix}) = \sqrt{(x- \ )^2 + (y- \ )^2}$



$y-(-3) = \sqrt{(x-4)^2+(y-1)^2}$

$(y+3)^2 = \left(\sqrt{(x-4)^2+(y-1)^2}\right)^2$

$y^2+6y+9 = (x-4)^2+(y-1)^2$

$\cancel{y^2}+6y+9 = x^2-8x+16+\cancel{y^2}-2y+1$

$+2y \qquad\qquad +2y$

$8y+9 = x^2-8x+16+1$

$-9 \qquad\qquad -9$

$\frac{8y}{8} = \frac{x^2-8x+8}{8}$

$y = \frac{x^2}{8} - \frac{8x}{8} + \frac{8}{8}$

$y = \frac{x^2}{8} - x + 1$

PROPORTIONAL OR DIRECT RELATIONSHIPS

Two variables, x and y, have a **direct (proportional) relationship** if for every ordered pair (x, y) we have:

$$\frac{y}{x} = k \quad \text{or} \quad y = kx$$

Stated succinctly, y will always be a constant multiple of x. The value of k is known as the **constant of variation**.

Exercise **#1:** In each of the following, x and y are directly related. Solve for the missing value.

(a) $y = 15$ when $x = 5$

$y = ?$ when $x = 9$

$$\frac{15}{5} = \frac{y}{9}$$

$135 = 5y$

$\boxed{y = 27}$

(b) $y = -6$ when $x = 4$

$y = ?$ when $x = -10$

$$-\frac{6}{4} = \frac{y}{-10}$$

$60 = 4y$

$y = 15$

(c) $y = 12$ when $x = 16$

$y = ?$ when $x = 24$

$$\frac{12}{16} = \frac{y}{24}$$

$288 = 16y$

$y = 18$

***Exercise* #2:** The distance a person can travel varies directly with the time they have been traveling if going at a constant speed. If Phoenix traveled 78 miles in 1.5 hours while going at a constant speed, how far will he travel in 2 hours at the same speed?

$\frac{\text{miles}}{\text{hour}}$

$$\frac{78}{1.5} = \frac{y}{2}$$

$$156 = 1.5y$$

$$y = 104 \text{ miles}$$

***Exercise* #3:** Jenna ~~works a job where her pay~~ varies ~~directly with the number of hours she has worked. In one~~ week, she worked 35 hours and made \$274.75. How many hours would she need to work in order to earn \$337.55?

$\frac{\text{Pay}}{\text{hours}}$

$$\frac{274.75}{35} = \frac{337.55}{x}$$

$$\frac{274.75x}{274.75} = \frac{11814.25}{274.75}$$

$$x = 43$$

43 hours

***Exercise* #4:** Two variables, x and y, vary directly. When $x = 6$ then $y = 4$. The point is shown plotted below.

(a) Find the y-values for each of the following x-values. Plot each point and connect.

$\frac{y}{x} = \frac{4}{6}$

$x = 3$

$\frac{4}{6} = \frac{y}{3}$

$12 = 6y$

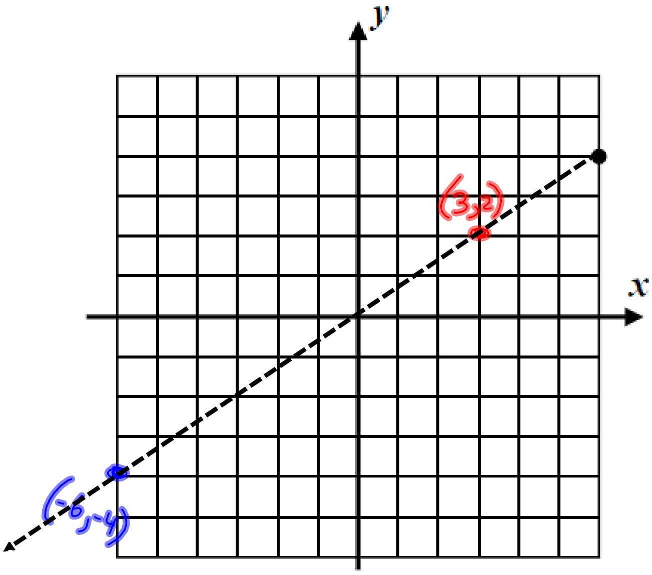
$y = 2$

$x = -6$

$\frac{4}{6} = \frac{y}{-6}$

$-24 = 6y$

$y = -4$



(b) What is the constant of variation in this problem? What does it represent on this line?

$\frac{y}{x} = \frac{4}{6} = \frac{2}{3}$ ← Slope of the line.

(c) Write the equation of the line you plotted in (a).

Need to know the y-value when x=0

$\frac{4}{6} = \frac{y}{0}$

$0 = 6y$

$y = 0$

$\boxed{y = \frac{2}{3}x + 0}$

Direct relationships often exist between two variables whose values are zero simultaneously.

***Exercise* #3:** The miles driven by a car, d, varies directly with the number of gallons, g, of gasoline used. Abagail is able to drive $d = 336$ miles on $g = 8$ gallons of gasoline in her hybrid vehicle.

(a) Calculate the constant of variation for the relationship $\frac{d}{g}$. Include proper units in your answer.

(b) Give a linear equation that represents the relationship between d and g. Express your answer as an equation solved for d.

(c) How far can Abagail drive on $g = 6$ gallons of gas?

(d) How many gallons of gas will Abagail need in order to drive 483 miles?